

Comparative study of basic digital modulation techniques: ASK, FSK and PSK

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ABSTRACT

This paper presents a comparison between the different basic digital modulation techniques which are amplitude shift keying, frequency shift keying and phase shift keying. The objective is to know the most optimal modulation technique in order to help telecommunication operators on the choice of modulation technique to be used. The method we have used is to make this comparison based on two essential criteria: the probability of error per symbol and the spectral efficiency. Using graphs of the error probabilities and signal spectra of the different modulations obtained in MATLAB, we have presented in tables the variations of these two criteria. The results show that the error probabilities of phase-shift keying and frequency-shift keying are small compared with amplitude-shift keying. The comparison of the spectral efficiency has therefore made it possible to distinguish between the two modulation techniques since the spectrum of the FSK signal remains lower than $\frac{1}{2}$ in contrast to phase-shift keying where the efficiency increases as a function of the number of symbols. A more practical simulation on an image transmission using these modulation techniques has made it possible to see more clearly the efficiencies of these modulation techniques. Thus, we can say that phase-shift keying is the optimal modulation.

Keywords: Telecommunication, transmission, modulation, probability of error, spectral efficiency.

I. INTRODUCTION

The field of telecommunications and networks is experiencing increasing a rapid change. It has undergone a revolution in the last two decades. We have for example the development of consumer computer systems [1]. Transmission networks have seen their capacity increase, including the throughput supported, the quality of transmission, etc. [2].

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However, the demand has remained ever greater. Indeed, in addition to traditional data, so-called multimedia content includes data such as high-quality sound and video and requires special attention for its transmission. The need for high-speed data transmission and appropriate technology is obvious [1].

Thus, several methods have emerged to remedy this constraint. In order to achieve significant performance over a transmission distance, wireless networks need to use techniques that will enable them to make this transmission possible. There are three types of basic digital modulations that are more frequently used [2], namely amplitude shift keying, frequency shift keying and phase shift keying. Subsequent studies have shown that FSK has a lower error probability than other types of modulation [3][4], however, this reasoning is based on the error probability criterion only. A good transmission is one that uses a reduced bandwidth [5], in order to reduce the spectral occupancy [6]. The aim of this work is therefore to make a comparative study between these digital modulation techniques in order to propose the optimal one. A simulation on MATLAB [7] will also be carried out in order to observe/simulate how this modulation is carried out in practice. The paper will be divided into two parts, first the methodology we use to make this comparison, the MATLAB simulations and finally the results and discussions.

II. METHODOLOGY

1. Principle of Digital Modulation

- The modulating signal

Indeed the modulating signal is the low frequency signal that contains the information to be transmitted. It is a discrete signal defined by [8]

$$c(t) = \sum_k c_k \cdot g(t - kT) = c_k(t) = a_k(t) + jb_k(t) \quad (1)$$

- The carrier signal

The carrier signal is the high-frequency signal, it is this signal that is carried. The following equation gives its representation:

$$S(t) = A \cos(\omega_0 \cdot t + \varphi) \quad (2)$$

- The modulated signal

Indeed, once the modulation technique is applied, a signal called modulated signal is obtained and is expressed by [8]

$$m(t) = \text{Re}[\sum_k c_k(t) \cdot e^{j(\omega_0 t + \varphi_0)}] \tag{3}$$

or

$$m(t) = \sum_k a_k(t) \cdot \cos(\omega_0 t + \varphi_0) - \sum_k b_k(t) \cdot \sin(\omega_0 t + \varphi_0) \tag{4}$$

Or more simply

$$m(t) = a(t) \cdot \cos(\omega_0 t + \varphi_0) - b(t) \cdot \sin(\omega_0 t + \varphi_0) \tag{5}$$

with $a(t) = \sum_k a_k(t)$ et $b(t) = \sum_k b_k(t)$

The signals $a(t)$ and $b(t)$ modulate respectively in phase and in quadrature using a rectangular pulse shape $g(t)$ called "formant": they are modulating trains and are written :

$$a_k(t) = a_k \cdot g(t - kT) \tag{6}$$

and

$$b_k(t) = b_k \cdot g(t - kT) \tag{7}$$

Each transmitted symbol corresponds to an elementary signal of the form:

$$m_k(t) = a_k \cdot g(t - kT) \cdot \cos(\omega_0 t + \varphi_0) - b_k \cdot g(t - kT) \cdot \sin(\omega_0 t + \varphi_0). \tag{8}$$

Fresnel decomposition in two-dimensional space with base vectors $g(t - kT) \cdot \cos(\omega_0 t + \varphi_0)$ and $-g(t - kT) \cdot \sin(\omega_0 t + \varphi_0)$ is shown in Figure 1.

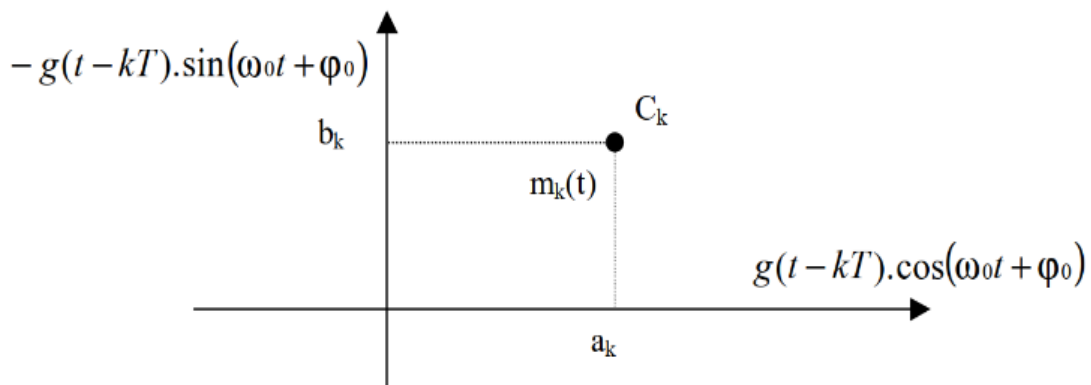


Figure 1: Position of a symbol in the Fresnel plane [8].

In formula 2, the parameters that can be changed are:

- The amplitude: A ;
- The frequency: $f_0 = \frac{\omega_0}{2\pi}$

- The phase: φ_0 .

A symbol is an element of an alphabet. If M is the size of the alphabet, the symbol is said to be M -ary. The symbol is binary for $M=2$. By grouping as a block, n independent binary symbol, we get an alphabet of $M = 2^n$. Thus an M -aire symbol conveys the equivalent of $n = \log_2 M$ [8].

2. Calculation of the probability of error

- Amplitude shift keying

The ASK is a monodirectional modulation and is carried out only on the carrier in phase $\cos(\omega_0 t + \varphi_0)$. The modulated signal is then written:

$$m(t) = \sum_k a_k \cdot g(t - kT) \cdot \cos(\omega_0 t + \varphi_0) \tag{10}$$

The error probability per symbol of the MDA is given by the relation:

$$P_s(e) = \frac{(M-1)}{M} \operatorname{erfc} \left(\sqrt{\frac{(3 \log_2 M) E_b}{(M^2-1) N_0}} \right) \tag{11}$$

And the error probability per bit by the relationship:

$$P_b(e) = \frac{P_s(e)}{\log_2 M} = \frac{P_s(e)}{k} \text{ où } k = \log_2 M \tag{12}$$

We get the graph below:

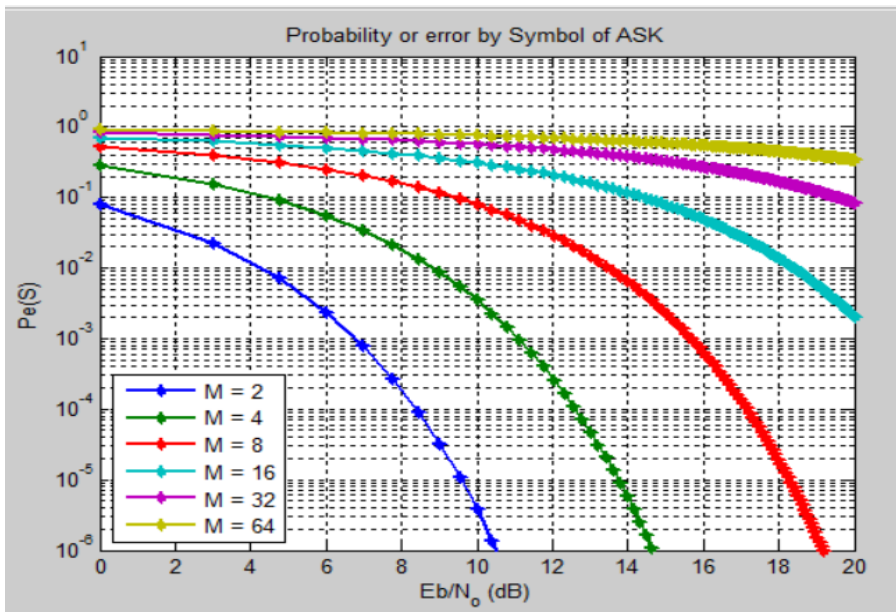


Figure 2: Probability of error per symbol MDA

- Frequency shift keying

The modulated signal can be written

$$m(t) = \text{Re}[e^{j\Phi(t)} \cdot e^{j\omega_0 t + \varphi_0}] \tag{13}$$

However, FSK have a constant envelope, i.e. $e^{j\Phi(t)} = \text{Cste}$. Thus, formula (10) is simplified by

$$m(t) = \cos(2\pi f_0 t + \Phi(t)) \tag{14}$$

The instantaneous frequency $f(t)$ of the signal $m(t)$ is obtained by deriving the phase $2\pi f_0 t + \Phi(t)$ in relation to time:

$$f(t) = f_0 + \frac{1}{2\pi} \frac{d\Phi}{dt} \tag{15}$$

Where f_0 and $\frac{1}{2\pi} \frac{d\Phi}{dt}$ represent the center frequency and the deviation respectively.

The error probability per symbol of the FSK is given by the relation

$$P_s(e) \leq \frac{1}{2} (M - 1) \text{erfc} \left(\sqrt{\frac{(\log_2 M) E_b}{2 N_0}} \right) \tag{16}$$

The plot of this probability according to the ratio $\frac{E_b}{N_0}$

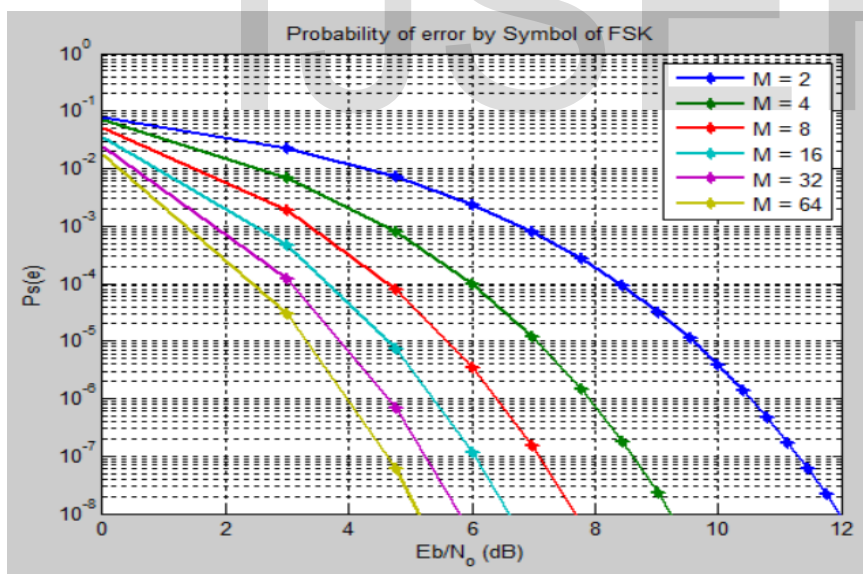


Figure 3: Probability of error per symbol of FSK

- **Phase shift keying (PSK)**

For the CDM, equation (1) is more simply written as

$$m(t) = A \cdot \cos(\omega_0 t + \varphi_0 + \varphi_k) \tag{17}$$

or

$$m(t) = A \cdot \cos(\omega_0 t + \varphi_0) \cdot \cos(\varphi_k) - A \cdot \sin(\omega_0 t + \varphi_0) \cdot \sin(\varphi_k) \quad (18)$$

It's directional modulation. The probability of error per symbol is given by the relation

$$P_s(e) = \text{erfc} \sqrt{\log_2 M \cdot \frac{E_b}{N_0} \cdot \sin \frac{\pi}{M}} \quad (19)$$

This probability is shown in Figure 4 below:

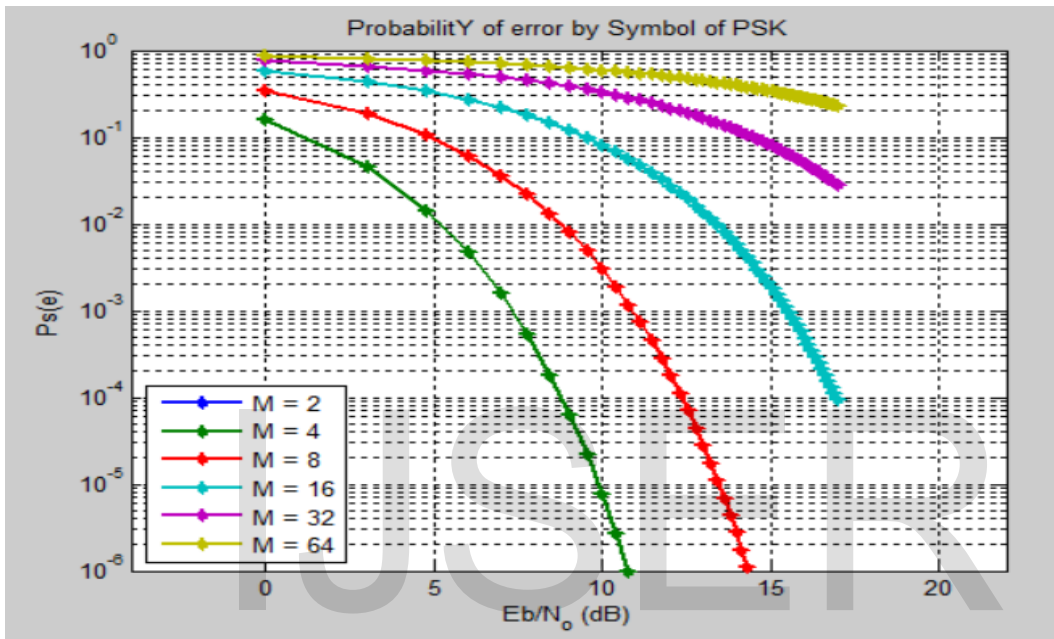


Figure 3: Probability of error per symbol of PSK

3. The spectral efficiency of the modulations

The spectral efficiency of a modulation measures the quality of bandwidth usage. Indeed, the more the bandwidth is reduced, the more perfect the transmission is. It results in

$$\eta = \frac{D}{B} \quad (20)$$

Where D is the bit rate and B is the bandwidth.

- Amplitude shift keying

In the spectrum of the ASK modulated signal (Figure 5) it can be seen that the carrier is surrounded by two sidebands corresponding to the data spectrum. The main lobe is thus 2D wide while the side lobes are half the width.

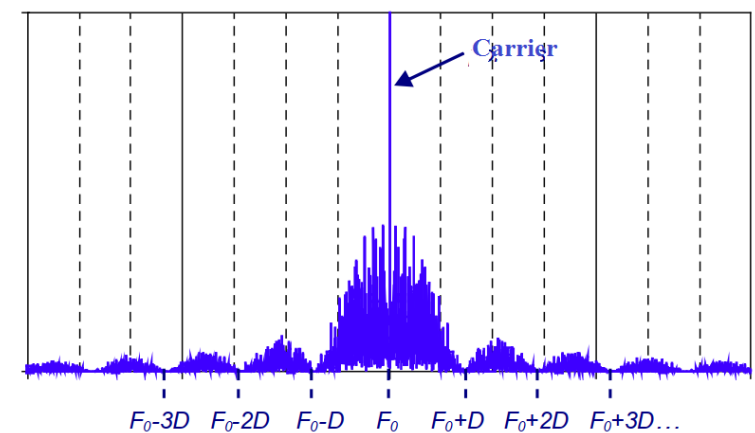


Figure 5: Spectrum of the raw ASK signal

After filtering, the spectrum is usually limited to the main lobe. The spectral clutter of the ASK modulated signal is of the order of 2 times the bit rate (Figure 6).

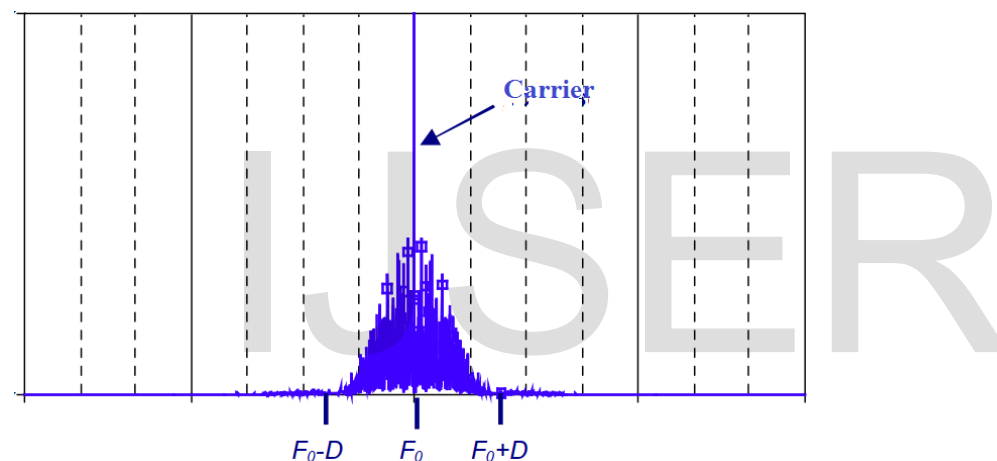


Figure 6: Spectre MDA filter

So we'll have $\eta = \frac{D}{B} = \frac{D}{2D} = \frac{1}{2}$

So the efficiency of amplitude shift keying modulations are of the order of 1/2.

- Frequency shift keying

The spectrum of an FSK signal is very complex. Indeed one finds an evolution in $\left| \frac{\sin x}{x} \right|$ around both frequency F_A et F_B . Everything happens as if we have two ASK spectra with the carrier frequency F_A et F_B (figure 7)

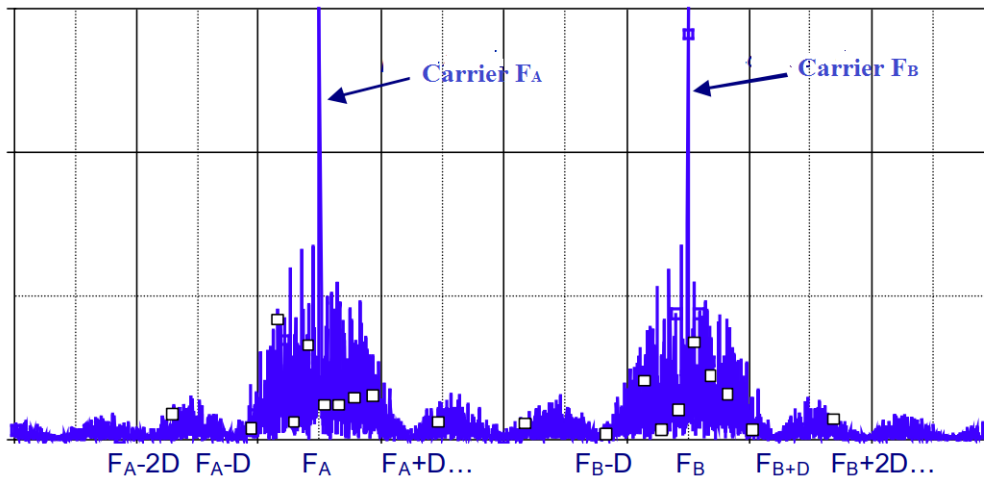


Figure 7: Unfiltered FSK spectrum

Regardless of the filtering performed, the spectrum extends from $F_A - D$ à $F_B + D$.

The bandwidth will be $B = (F_B + D) - (F_A - D) = F_B - F_A + 2D$ or $B = 2\Delta F + 2D$

So we'll have: $\eta = \frac{D}{B} = \frac{D}{2\Delta F + 2D} < \frac{1}{2}$ FSK have spectral efficiencies that are difficult to obtain from $\frac{1}{2}$.

- **Phase Shift Keying**

The carrier has a constant amplitude and a phase at the origin which can take the two values 0 and π . The following figure shows the spectrum of 2-PSK modulation.

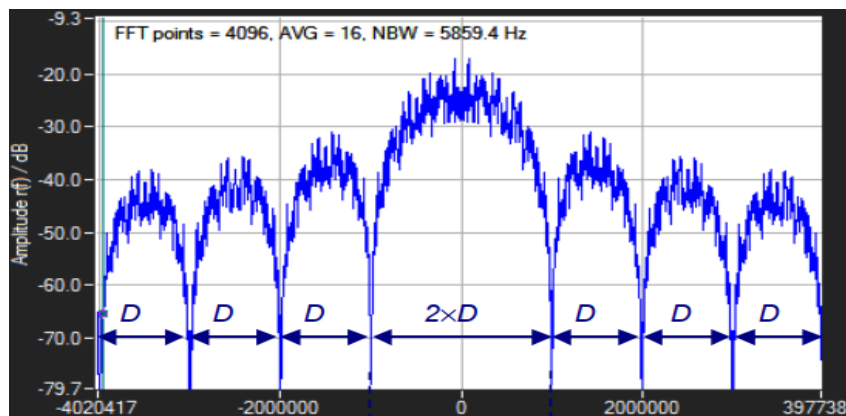


Figure 8: 2-PSK spectrum

In Figure 8, the central lobe is twice the width of the bit rate D and the side lobes are half the width.

After filtering, only the centre lobe is retained (see Figure 9).

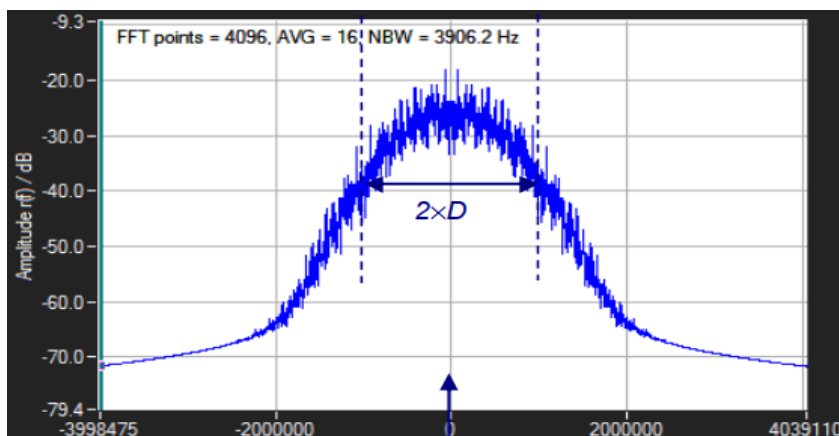


Figure 9: Filtered 2-PSK spectrum

So after filtering we have: $B = 2D$

This results in $\eta = \frac{D}{B} = \frac{D}{2D} = \frac{1}{2}$

So we have a spectral efficiency of $\frac{1}{2}$.

For $M=4$, i.e. for 4-PSK, we have the following spectrum:

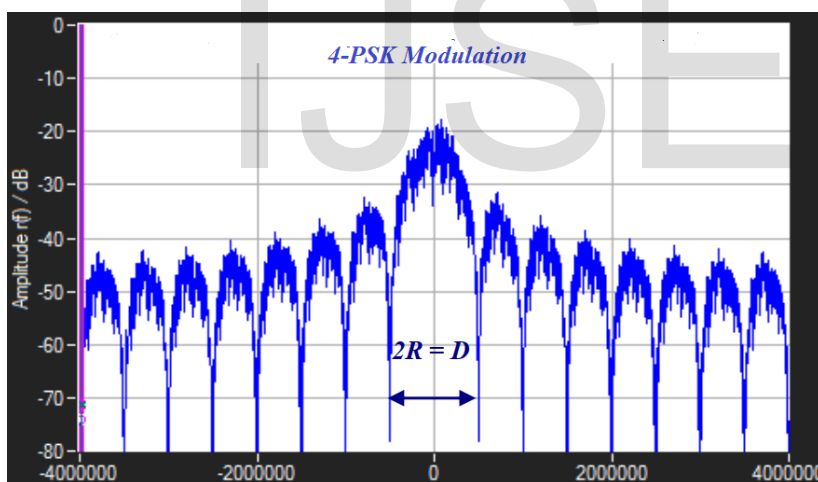


Figure 10: 4-PSK spectrum

Figure 10 shows that the width of the strip is D . The spectral efficiency is therefore equal to 1.

For M -ary PSK, we can see that the spectral efficiency increases as M increases. The following table shows this increase.

Table 1: Spectral efficiency of M-PSK

n	M	Modulation	Flow	Efficiency
1	2	2-PSK	D	η
2	4	4-PSK	2. D	$2.\eta$
3	8	8-PSK	3. D	$3.\eta$

III. RESULTS

1. Probability of error per symbole

- Probability of error of ASK, FSK and PSK for SNR=5

The following table gives the values of the error probabilities per symbol obtained from the three types of modulation for different values of M

Table 2: Values of the error probabilities per symbol for SNR=5

M	MDA	MDF	MDP
2	7.10^{-4}	$1, 2.10^{-3}$	15.10^{-4}
4	34.10^{-3}	38.10^{-3}	15.10^{-4}
8	20.10^{-2}	88.10^{-3}	36.10^{-3}
16	46.10^{-2}	19.10^{-2}	21.10^{-2}

- Probability of error of ASK, FSK and PSK for SNR=10

Table 3: Values of the error probabilities per symbol for SNR=10

M	MDA	MDF	MDP
2	3.10^{-6}	$7, 8.10^{-4}$	$7, 7.0^{-6}$
4	$3, 5.10^{-3}$	23.10^{-4}	$7, 7.10^{-6}$
8	79.10^{-3}	54.10^{-4}	30.10^{-4}
16	31.10^{-2}	11.10^{-3}	80.10^{-3}

- **Probability of error of ASK, FSK and PSK for SNR=15**

Table 4: Values of the error probabilities per symbol for SNR=15

M	MDA	MDF	MDP
2	2.10^{-8}	$5, 3.10^{-5}$	$4, 3.0^{-8}$
4	3.10^{-4}	$1, 6.10^{-4}$	$4, 3.10^{-8}$
8	33.10^{-3}	$3, 7.10^{-4}$	$2, 8.10^{-4}$
16	22.10^{-2}	80.10^{-3}	32.10^{-3}

2. Les efficacités spectrales

Table 5: Spectral efficiency of ASK, FSK and PSK

Modulation	Efficacité
MDA	1/2
MDF	<1/2
MDP-2	1/2
MDP-4	1
MDP-8	3/2
MDP-M	$\text{Log}_2 M \times 1/2$

It can be seen that for DA and FSK, the spectral efficiencies are constant, while for CDM, the spectral efficiency increases to $\log_2 M \times 1/2$.

3. Transmission of an image

The following figure shows the result of a MATLAB simulation of the transmission of an image.



Figure 10: Image modulation and demodulation

The figure above confirms the results of the different comparison tables. Indeed the images obtained by the PSK modulations are better than the others.

IV. CONCLUSION

Our work consisted in making a comparison between the three basic modulation techniques in order to provide telecommunications operators with the most optimal technique. The results show that phase-shift keying offers a good probability of error per bit or per symbol. Also, this same technique has the advantage of having a much better spectral efficiency than the other techniques.

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